தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்
ஐந்தாம் தவணைப் பரீட்சை - 2022

FWC

# Conducted by Field Work Centre, Thondaimanaru
## Fifth Term Examination – 2022
# Combined Mathematics I

**Grade : 13(2022)** | **10 E I** | **Time : Three hours** **Additional Reading Time – 10 minutes**

**Index No.** | | | | | | |

**Instructions:**

- **Part A** (Questions 1 - 10) and **Part B** (Questions 11 - 17).
- **Part A :**
  Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space needed.
- **Part B :**
  **Answer five** questions only.
- At the end of the time allotted, tie the answer scripts of the two pats together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- You are permitted to remove only Part B of the question paper from the Examination Hall.

**For Examiners' Use only**

| (10) Combined Mathematics - I | | |
|---|---|---|
| **Part** | **Question No.** | **Marks** |
| A | 1 | |
| | 2 | |
| | 3 | |
| | 4 | |
| | 5 | |
| | 6 | |
| | 7 | |
| | 8 | |
| | 9 | |
| | 10 | |
| B | 11 | |
| | 12 | |
| | 13 | |
| | 14 | |
| | 15 | |
| | 16 | |
| | 17 | |
| **Total** | | |

| | |
|---|---|
| **Combined Maths-I** | |
| **Combined Maths-II** | |
| **Final Marks** | |

## Part A

1. Using the **Principle of Mathematical Induction,** prove that $\sum_{r=1}^{n}(-1)^{r-1}r^2 = (-1)^{n-1}\frac{n}{2}(n+1)$ for all $n \in \mathbb{Z}^+$.

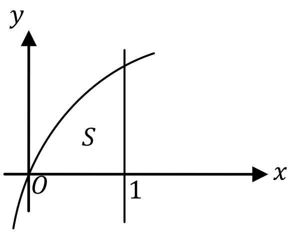
2. Sketch the graphs of $y = |x^2 - 1|$ and $y = x + 1$ in the same diagram. **Hence or otherwise,** find all real values of $x$ satisfying the inequality $|x^2 - 1| \leq x + 1$.

3. Sketch, in an Argand diagram, the locus of the points that represent complex numbers $z$ satisfying $|z - i| = |z - 1|$. **Hence,** find the minimum value of $\left|\frac{z+i}{1+i}\right|$ satisfying $|iz - i| = |iz + 1|$.

4. Show that $\lim_{x \to \frac{\pi}{2}} \frac{1-\sqrt{\sin x}}{(2x-\pi)^2} = \frac{1}{16}$.

5. Show that $\frac{d}{dx}\{x\ln(\sqrt{x-1}+\sqrt{x+1})\} = \frac{x}{2\sqrt{x^2-1}} + \ln(\sqrt{x-1}+\sqrt{x+1})$.
**Hence,** find $\int \ln(\sqrt{x-1}+\sqrt{x+1})\ dx$.

6. Let $S$ be the region enclosed by the curves $y = \frac{x}{\sqrt{x^2+1}}$, $y = 0$ and $x = 1$. Show that the area of $S$ is $\sqrt{2}-1$ square units. The region $S$ is rotated about the $x$-axis through $2\pi$ radians. Show that the volume of the solid thus generated is $\frac{\pi}{4}(4-\pi)$ cubic units.



7. Show that the equation of the normal line to the rectangular hyperbola parametrically given by $x = \frac{c}{t}$ and $y = ct$ for $t \neq 0$ and $c \neq 0$, at the point $\left(\frac{c}{t},\ ct\right)$ is given by $t^3y - tx = c(t^4 - 1)$. The normal line to this hyperbola at $P\left(\frac{c}{2},\ 2c\right)$ meets the hyperbola again at another point $Q\left(\frac{c}{T},\ cT\right)$. Show that $8T^2 - 15T - 2 = 0$

8. Let $A(-2, 0)$ and $B(1, 4)$. Show that the coordinates of any point on the line $l$ through the point $B$ and perpendicular to $AB$ are given by $(1 - 4t, 4 + 3t)$, where $t \in \mathbb{R}$. Find the coordinates of the pints $P$ and $Q$ on $l$ such that $BP = BQ = 5$ units.

9. Show that the straight line $x + 3y - 6 = 0$ intersects the circle $x^2 + y^2 - 2x - 4y - 4 = 0$ at two distinct points. Find the equation of the circle on the line segment joining these points as a diamerer.

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10. Express $2\sin x\cos x + 2\sin^2 x - 1$ in the form $R\sin(2x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$. **Hence**, solve the equation $\sin x\cos x + \sin^2 x = 1$.



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தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்

# Conducted by Field Work Centre, Thondaimanaru

## Fifth Term Examination – 2021

## Combined Mathematics I

## Part B

FWC



11. (a) Let $f(x) = x^2 - (\lambda + 2)x + (2\lambda - 1)$ for $\lambda \in \mathbb{R}$. Show that the equation $f(x) = 0$ has two distinct real roots.

Let $\alpha$ and $\beta$ be the roots of the equation $f(x) = 0$. Write down $\alpha + \beta$ and $\alpha\beta$ in terms of $\lambda$, and find the values of $\lambda$ such that both $\alpha$ and $\beta$ are positive.

Show also that the quadratic equation whose roots are $\alpha^2$ and $\beta^2$ is $x^2 - (\lambda^2 + 6)x + (2\lambda - 1)^2 = 0$, and **deduce** that the quadratic equation whose roots are $1 + \alpha^2$ and $1 + \beta^2$ is $x^2 - (\lambda^2 + 8)x + 5\lambda^2 - 4\lambda + 8 = 0$.

(b) Let $f(x) = 4x^3 + 5x^2 + ax + b$ and $g(x) = x^3 + cx + 2$, where $a, b, c \in \mathbb{R}$. It is given that $x - 1$ and $x + 2$ are factors of $f(x)$. It is also given that $g(x) = (x-1)^2\phi(x)$, where $\phi(x)$ is a lenear function in $x$. Find the values of $a, b$ and $c$.

For these values of $a, b$ and $c$, show that $f(x) - 4g(x) = 5x^2 + 5x - 10$.

Also show that $f(x) - 4g(x) \geq -\frac{45}{4}$ and the remainder when $f(x) - 4g(x)$ is diveded by $(x+2)^2$ is $-15(x+2)$.

12. (a) It is required to form 4-digit number consisting of 4 digits taken from the 15 digits given below:
1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5
Find the number of different such 4-digit numbers that can be formed

(i) if all 4 digits chosen are different,

(ii) if all 4 digits chosen are different and the 4-digit number is divisible by 3,

(iii) if any 4-digits can be chosen.

(b) Let $U_r = \frac{9r^3 + 21r^2 + 13r - 1}{(3r-1)^2(3r+2)^2}$ for $r \in \mathbb{Z}^+$.

Determine the values of the real constants $A$ and $B$ such that $U_r = \frac{A\,r}{(3r-1)^2} - \frac{r+B}{(3r+2)^2}$ for $r \in \mathbb{Z}^+$.

**Hence,** find $f(r)$ such that $\left(\frac{1}{2}\right)^{r+1} U_r = f(r) - f(r+1)$ for $r \in \mathbb{Z}^+$. and show that $\sum_{r=1}^{n} \left(\frac{1}{2}\right)^{r+1} U_r = \frac{1}{8} - \frac{n+1}{(3n+2)^2}\left(\frac{1}{2}\right)^{n+1}$ for $n \in \mathbb{Z}^+$.

**Deduce** that the infinite series $\sum_{r=1}^{\infty} \left(\frac{1}{2}\right)^{r+1} U_r$ is convergent and find its sum.

Also find $\sum_{r=2}^{\infty} \left(\frac{1}{2}\right)^{r} U_r$.

13. (a) Let $z_1 = 2(\sqrt{3} + i)$ and $z_2 = 2(1 - i)$. Express $\frac{z_1}{z_2}$ in the form $x + iy$, where $x, y \in \mathbb{R}$. Also express each of the complex numbers $z_1$ and $z_2$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $0 < \theta < \frac{\pi}{2}$.

**Hence**, show that $\frac{z_1}{z_2} = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$. **Deduce** that $\sin\left(\frac{5\pi}{12}\right) = \frac{\sqrt{3}+1}{2\sqrt{2}}$.

(b) Let $z \in \mathbb{C}$. Show that $z = \bar{z}$ **if and only if** $z$ is a real number.

**Hence or otherwise** show that If $\frac{z}{1+z^2}$ is a real number, $|z| = 1$, where $z \in \mathbb{C}$ and $z \notin \mathbb{R}$.

(c) Let $\theta \in \mathbb{R}$ and $z = \cos\theta + i\sin\theta$. Show that $\frac{1}{z} = \cos\theta - i\sin\theta$. **Deduce** that $z + \frac{1}{z} = 2\cos\theta$.

**Hence,** if $x + \frac{1}{x} = 2\cos\theta$, show that $x = \cos\theta + i\sin\theta$ or $x = \cos\theta - i\sin\theta$ for $\theta \in \mathbb{R}$.

**Using the above results** and **De Moivre's theorem,** If $x + \frac{1}{x} = 2\cos\theta$, show that $x^n + \frac{1}{x^n} = 2\cos n\theta$, where $\theta \in \mathbb{R}$ and $n \in \mathbb{Z}^+$ and **deduce that** $\frac{x^{2n}+1}{x^{2n-1}+x} = \frac{\cos n\theta}{\cos(n-1)\theta}$.

14. (a) Let $f(x) = \frac{(x+2)^2}{(x+3)^3}$ for $x \neq -3$.

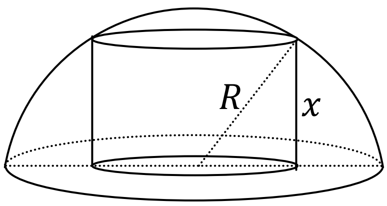
Show that $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = -\frac{x(x+2)}{(x+3)^4}$ for $x \neq -3$.

**Hence,** find the interval on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing. Also, find the coordinates of the turning points of $f(x)$.

Sketch the graph of $y = f(x)$ indicating the asymptotes, the turning points and y-intercept.

**Hence,** sketch the graph of $y = -f(x)$.

(b) A solid rectangular cylinder with an axis passing through the centre of the sphere is cut from a solid hemisphere of radius $R$ as shown in the figrue. Axes of both coincide with each other. Let the height of the cylinder be $x$. Show that the volume $V$ of the cylinder is given by $V = \pi(R^2x - x^3)$ for $0 < x < R$.




Show that the volume of the cylinder cannot exceed $\frac{1}{\sqrt{3}}$ times the volume of the hemisphere.

15. (a) Find the values of the constants $A$, $B$ and $C$ such that

$16x^4 + 4x^3 + 16x^2 + x + 1 \equiv A(4x^2+1)^2 + Bx(4x^2+1) + Cx^2$ fo all $x \in \mathbb{R}$.

Hence, write down $\frac{16x^4+4x^3+16x^2+x+1}{x(4x^2+1)^2}$ in partial fractions and find $\int \frac{16x^4+4x^3+16x^2+x+1}{x(4x^2+1)^2}\,dx$

(b) Using the substitution $t = \sqrt{x}$, show that $\int_0^1 \frac{x^{\frac{3}{2}}}{1+x}\,dx = 2\int_0^1 \frac{t^4}{1+t^2}\,dt = \frac{1}{6}(3\pi - 8)$.

Using integration by parts, show that $\int_0^1 x\tan^{-1}\sqrt{x}\,dx = \frac{\pi}{8} - \frac{1}{4}\int_0^1 \frac{x^{\frac{3}{2}}}{1+x}\,dx$ and find $\int_0^1 x\tan^{-1}\sqrt{x}\,dx$.

(c) Using the formula $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$, where $a$ and $b$ are constants, show that $\int_{-1}^1 \frac{x^{2022}}{1+e^x}\,dx = \int_{-1}^1 \frac{x^{2022}e^x}{1+e^x}\,dx$ and find $\int_{-1}^1 \frac{x^{2022}}{1+e^x}\,dx$.

**Deduce** that $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{x^{2022}}{1+e^{2x}}\,dx = \frac{1}{2023\times 2^{2023}}$.

16. Prove that the perpendicular distance from the point $P \equiv (x_1, y_1)$ to the line $ax+by+c=0$ is $\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$.

Show that the point $A \equiv (-1, 2)$ lies on the straight line $l \equiv x - 2y + 5 = 0$. Also, show that the coordinates of any point on the line $l \equiv x - 2y + 5 = 0$ are given by $(2t-1, t+2)$, where $t \in \mathbb{R}$.

Find the equations of two circles $s_1$ and $s_2$ having centre at the line $l$ and having radius $\sqrt{10}$ units and touching the line $l_1 \equiv 3x - y + 5 = 0$ through $A$. Find the equation of the other common tangent through $A$ to $s_1$ and $s_2$.

Find the equation of the circle with centre $A$ and bisecting the circumference of the two circles $s_1$ and $s_2$.

17. (a) Write down $\sin(A+B)$ in terms of $\sin A$, $\cos A$, $\sin B$ and $\cos B$.

**Hence,** prove that $\sin 2A = 2\sin A\cos A$ and $\cos 2A = 1 - 2\sin^2 A$.

**Using the above results,** show that $\sin 3A = 3\sin A - 4\sin^3 A$ and **deduce** that $\cos 3A = 4\cos^3 A - 3\cos A$.

Solve the equation $\cos 3x - \sin 3x - 3(\sin x + \cos x) = 0$.

(b) In the usual notation, state the **Cosine Rule** for a triangle $ABC$.

Show that $\frac{\cos A}{a} + \frac{\cos B}{b} = \frac{c}{ab}$.

**Deduce** that $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2+b^2+c^2}{2abc}$.

(c) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, show that $x^2 + y^2 + z^2 + 2xyz = 1$.

**Deduce** that $\sin^2 A + \sin^2 B + \sin^2 C + 2 \sin A \sin B \sin C = 1$, where $A$, $B$ and $C$ are acute angle and $A + B + C = \frac{\pi}{2}$.

# தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்
## ஐந்தாம் தவணைப் பரீட்சை - 2022

# Conducted by Field Work Centre, Thondaimanaru.
# 5th Term Examination - 2022

FWC

| Grade :- 13 (2022) | Combined Mathematics II - A | Three hours and ten minutes |
|---|---|---|

Admission No | | | | | |

**Instructions**

- This question paper consists of two parts; Part A (questions 1 - 10) and part B (questions 11 - 17).

**Part - A**

- Answer all questions. Answers should be written in the space provided on the questions paper. If additional space needed, you may use additional answer sheets.

**Part - B**

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- Only part B of the paper is allowed to be taken out of the examination hall.

| Combined mathematicsI | | |
|---|---|---|
| **Part** | **Question** | **Marks** |
| A | 1 | |
| | 2 | |
| | 3 | |
| | 4 | |
| | 5 | |
| | 6 | |
| | 7 | |
| | 8 | |
| | 9 | |
| | 10 | |
| B | 11 | |
| | 12 | |
| | 13 | |
| | 14 | |
| | 15 | |
| | 16 | |
| | 17 | |
| | **Total** | |

| | |
|---|---|
| **Combined Maths I** | |
| **Combined MathsII** | |
| **Total** | |
| **Final Marks** | |

## Part – I A

1) Particles P, Q having masses m, M are moving with velocities 4u, u on a smooth horizontal table at a certain distance between them. Particles move with velocities of $4u, u$ respectively in $\overrightarrow{PQ}$ direction and collide directly. Impulse caused by collision is $5mu$ and the coefficient of restitution between particles is $\frac{4}{5}$. Find the velocities of particles P and Q just after collision and Find M in terms of m.

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2) A particle is projected in vertical plane with a velocity of $4g\boldsymbol{i} + 3g\boldsymbol{j}$ from a point O at a height $8g\ m$ from horizontal ground. It travels under the action of gravity and reaches horizontal ground at point A. Find the maximum height achieved by particle above horizontal ground and find the velocity of particle when hitting the horizontal ground.



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3) Particles of masses of 2m and m are attached to ends of a light inextensible string of length $4a$ passing over smooth light pulley fixed at a height of $4a$ from horizontal ground. System is released slowly such that both the particles are in same horizontal level and the portions of strings are taut and vertical. Find the accelerations of particles by using principle of conservation of energy when particle of mass 2m is at a depth of $x$ from pulley. Find the speed of particle having mass 2m with which it reaches the horizontal ground.

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4) Mass of engine of a train is $100kg$. When this engine travels with a velocity of $V\ m\,s^{-1}$, The resistive force is found to be $3V^2$ and the power exerted by the engine is found as $\frac{3000}{V}\ W$.

i) Show that the tractive force provided by the engine is 120 N when the train travels with a velocity of $5\ m\,s^{-1}$ on a horizontal land and calculate the acceleration of engine.

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ii) Find the speed of engine when it travels with a uniform velocity upwards an inclination of $\sin^{-1}\left(\frac{1}{98}\right)$.

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5) Ends of a light inextensible string of length $25a$ are joint to fiexd points A and B. A lies vertically above B. A smooth ring of mass m is attached to string and the ring is moving in a circle with a constant velocity having B as the center. The part of the string joining A and the ring makes an angle of $\tan^{-1}\left(\frac{5}{12}\right)$ with horizontal. Find the,

i) Tension in the string

ii) Velocity of ring.

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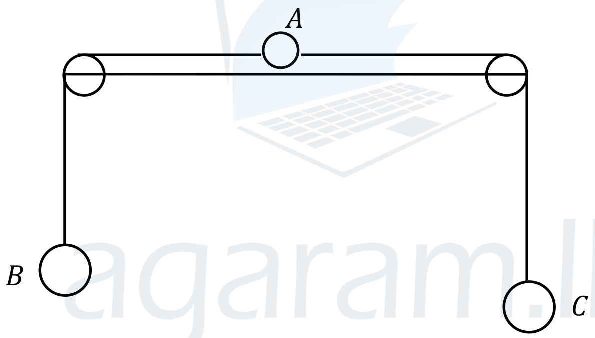


Three particles A, B and C of masses m, 2m and 4m are attached to ends two light inextensible strings, as shown in the figure. 'A' lies on a rough table and the strings pass over two smooth light pulleys fixed at the ends of the table and hold the particles B and C. The system is released slowly as the parts of the strings are taut. Obtain sufficient equations to find the tensions in strings and coefficient of friction between particle A and table. Find the co-efficient of friction. Where, acceleration of particle B is $^{g}/_{4}$.

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7) Position vectors of points A and B with respect to origin O are $2\sqrt{3}\boldsymbol{i} + 2\boldsymbol{j}$ , $\sqrt{3}\boldsymbol{i} - \boldsymbol{j}$ respectively. Find the position vector of point C such that OACB is a parallelogram. Find the position vector of point D on OA such that OC is perpendicular to BD.

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8) One end A of a uniform rod AB of weight $W$ and length $4a$ is hinged to a fixed point smoothly. One end of light inextensible string of length $6a$ is attached to a point C on the rod such that $AC = 3a$ and its other end is joint to a point D which is vertically above A. The rod is in equilibrium horizontally. at equilibrium, find the tension in the string and reaction in the hinge .



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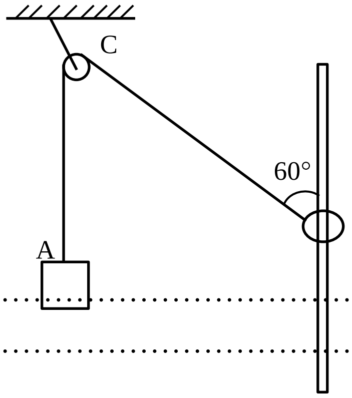
9) A light inextensible string passes over a fixed smooth light pulley C. End A of the string is attached to a weight of $4w$ and end B is joint to a ring of weight $w_1$ which is attached to a rough straight rod fixed vertically as shown in the figure. If the ring is just about to slip downwards while BC makes an angle of $60°$ with upward vertical, find the value of $w_1$. Coefficient of friction between rod and ring is $\frac{\sqrt{3}}{2}$




10) A,B are two events of a sample space such that $P(A) = \frac{1}{2}$ , $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{4}$.

Find (i) $P(A/B)$

(ii) $P(B/A)$

(iii) $P(A'/B)$

தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்
ஐந்தாம் தவணைப் பரீட்சை - 2022
Conducted by Field Work Centre, Thondaimanaru.
5th Term Examination - 2022
FWC

**Grade :- 13 (2022)** **Combined Mathematics II - B**

11)

(a) There are two straight parallel railway tracks running through a station O. Two trains A and B pass the station O in the same direction at the same time along those two tracks. Train A has a velocity '$u$' when passing the station O and it travels with a constant acceleration '$f$'. Train B starts from rest at station O and travels with a constant acceleration of '$2f$'. After travelling for time '$T$', both trains reach same velocity. Train B travel with the constant velocity after passing Train A. Just after Train B passes Train A, Train A increases its acceleration to '$2f$' and passes Train B

(i) Draw the V-T graph for both trains in same axis until train A passes train B.

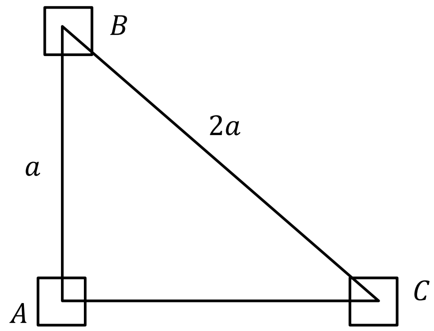
(ii) Show that $T = \frac{u}{f}$

(iii) Find the time taken for B to pass A initially, in terms of $T$.

(iv) Find the velocities of A and B when B passes A initially.

(v) Find the velocity of A when A passes B again.

(b) Three cyclists A, B and C are on a level ground. At a particular moment, B is at $a$ km due North to A and C is at due East of A. Where BC=2a. They start their journey at the same time. A travel with a uniform velocity at 30° East of North, B travels with uniform velocity $\sqrt{3}u$ at 30° East of South. C travels with uniform velocity $u$. For 'C', 'A' seems to be travelling with a velocity $2u$ in the direction of 60° East of North




(i) Draw the path of A relative to C

(ii) Find the shortest distance between A and C

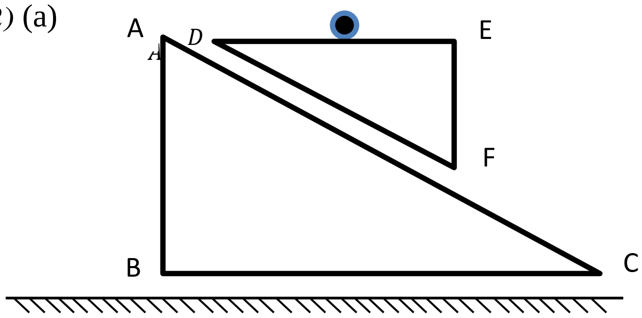
(iii) Find the velocity of A relative to Earth by drawing a velocity triangle, using relative velocity principle.

(iv) Using relative velocity principle, find the velocity of A relative to B

(v) Show that A will meet B and find the time taken for it.

(vi) Find the direction of motion of C

*12)* (a)




In the figure, right angled triangles ABC and DEF are vertical cress-sections through the center of gravity of uniform wedges of mass 5m and 3m respectively. Here $A\hat{B}C = D\hat{E}F = 90°$ and $B\hat{C}A = 30°$. Wedge ABC is kept such that its face BC lies on smooth horizontal floor. The other wedge DEF is kept as face DF touching the line of greatest slope of face AC and Face DE is horizontal as shown in the figure. A particle P of mass m is kept on surface DE and the system is released from rest gently. Obtain sufficient equations to determine the accelerations of wedges and particle P. Write the equation to find the reaction exerted by wedge DEF on particle P

(b) One end of light inextensible string of length $2l$ is attached to a point O at a particular height and a particle $P$ of mass $m$ is attached to the other end. $P$ is kept at a distance of $\sqrt{3}\,l$ at the level of O and released gently.

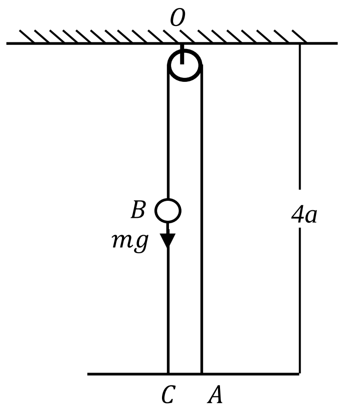
(i) Find the velocity of particle just before the string become tight.

(ii) Find the velocity of particle just after the string is taut

(iii) Find the velocity of the particle and the tension in the string when the string makes an angle of $\theta$ with downward vertical such that $0 < \theta < \frac{\pi}{3}$

(iv) Deduce the depth from O at which the particle attain Instantaneously rest for the first time.

*13)* One end of a light elastic string AB of natural length $2a$ and modulus of elasticity $\lambda$ is attached to a point A on the ground and the string passes over a light smooth pulley fixed at a height of $4a$ above the horizontal ground and a particle of mass m is attached to the other end B. One end of another light string of natural length a and modulus of elasticity $\lambda$ is attached to a point C on the horizontal ground and the other end is attached to the same particle as shown in the figure and the particle is in equilibrium at a depth of $2a$ below O. At equilibrium the particle is pulled upward a distance of $a$ and released slowly.



(i) Show that elastic modulus $\lambda = mg$

(ii) Show that the particle satisfy the equation of motion $\ddot{x} = \frac{-3g}{2a}(x - 2a)$ when it is at a depth of $x\ (x < 3a)$ below O.

(iii) Show that the motion of particle is a Simple Harmonic Motion and find the center of oscillation

(iv) It is given that the solution of the equation of motion is in the form of $\dot{x}^2 = \omega^2(b^2 - X^2)$. Find $\omega$ and $b$. Where $X = x - 2a$

(v) Calculate the speed of particle at $x = 3a$

(vi) If the string BC is cut off when the particle is at the lowest point below O, Show that the particle satisfy the equation of a simple harmonic motion $\ddot{y} = \frac{-g}{2a}y$ when the particle is at a depth $y$ below O and write the center of oscillation and amplitude.

(vii) Find the total time taken by the particle to reach the pulley since it has started its motion

*14)*

(a) Position vectors of A, C with respect to O are $\boldsymbol{a}, \boldsymbol{a} + \boldsymbol{b}$ respectively. Position vector of D is $\boldsymbol{3a}$. Extended DC meets the line drawn parallel to AC through O at M. Line drawn parallel to AO through C meets OM at B. Line drawn parallel to AB through O meets extended CB at N. Let $DM = \lambda\, DC, OM = \mu\, OB$.

(i) Find the position vector of B

(ii) Find the position vector of N

(iii) Find $\overrightarrow{DC}$

(iv) $\overrightarrow{DM}, \overrightarrow{OM}$n terms of $\lambda, \mu, a, b$

(v) Find $\lambda, \mu$ by using suitable vector addition.

(vi) Deduce OB:BM

.

(b) ABCD is a trapezium such that $AB \parallel DC$ and M is a point on AB such that

$\frac{1}{2}AD = DC = AM = \frac{1}{4}MB$. Where $A\hat{B}C$ = 30°. Forces of $5P, \sqrt{3}P, \lambda P, 3P, 2P$ and $4P$ Newtons act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{DC}$, $\overrightarrow{CM}$, $\overrightarrow{AD}$. .

(i) Find $\lambda$ if the resultant in parallel to MD.

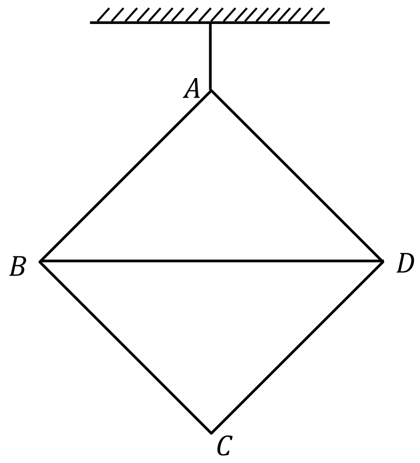
(ii) Find the magnitude of the resultant

(iii) Find the distance from M to the point at which line of action of resultant intersects AB.

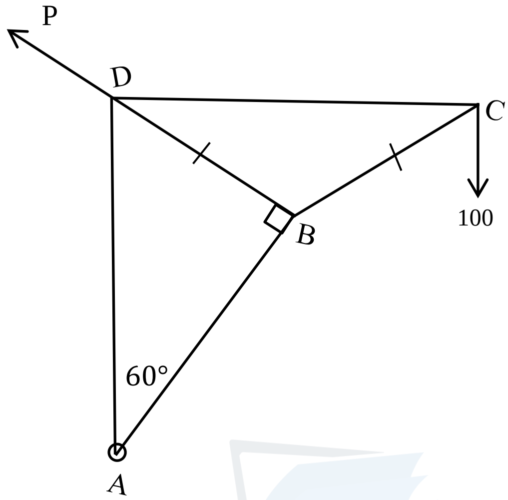
(iv) Find the couple that should be added to make the resultant go through C.

15)

(a) Two uniform rods AB and AC, each of length $2\sqrt{3}a$ and weight , another two uniform rods BC and CD, each of length $3\sqrt{2}a$ and weight $2w$, are smoothly hinged at the ends as shown in the figure. The system is hung from A and kept at equilibrium by joining a light rod of length $6a$ to BD at the joints B and D. At equilibrium, Find the reaction at C and the thrust in rod BD




(b)




The figure shows a framework made by joining light rods AB, BC, CD, BD and AD. It is smoothly hinged at A and kept at equilibrium such that AD is vertical and DC is horizontal, by applying a force $P$ along $\overrightarrow{BD}$ and hanging a weight of 100 N at C

(i) Draw a stress diagram using Bow's notation and distinguish the stresses in rods whether tension or thrust and find their magnitudes.

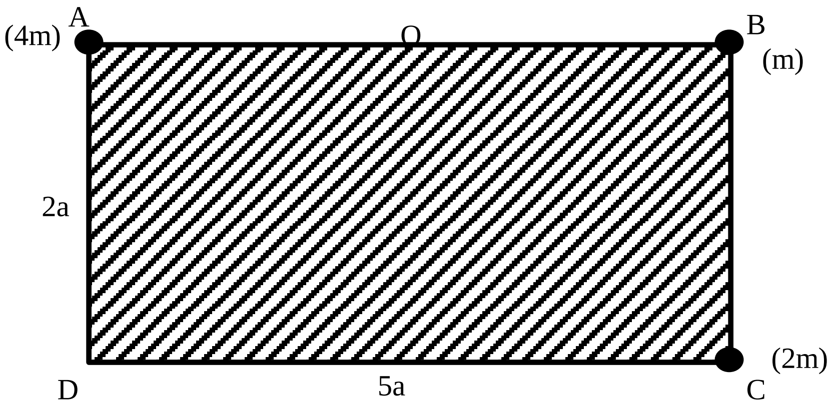
(ii) Hence the stress diagram, find the value of $P$ and reaction at A

16)

(a) Using integration, find the center of mass of a uniform rod of length $2a$

(b) Using integration, find the center of mass of a uniform rectangular lamina of length $2a$ and breadth $2b$

(c) ABCD is a uniform rectangular lamina such that AD=2a, DC=5a. Mass of rectangular lamina is $3m$. Masses of $4m, m$ and $3m$ are attached to vertices A, B and C respectively.



(i) Calculate the distance to the center of mass of system from AD and AB

(ii) O is the midpoint of AD. The system is hanged freely from O. Find the angle made by AB with horizontal

(iii) By applying a horizontal force $P$ at C along CD, the system hanged at O is brought to equilibrium such that AB horizontal and vertex C hanging vertically below B. Show that P=5/4mg

*17)*

(a) If A,B are two events of a sample space, then prove that

(i) $P(A) = P(A \cap B) + P(A \cap B')$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(iii) $P(A'/B) = 1 - P(A/B)$

(b) A box contains 4 red marbles and 8 green marbles. A marble is taken at random and the colour of the marble is noted. Then, that marble is placed into the box again along with 2 more marbles of the same colour. Marbles are taken thrice in the same manner.

(i) Find the probability of getting red marble in all three attempts.

(ii) Find the probability of getting marbles in the order of red, green, red in three attempts

(iii) what is the probability of getting a green marble in the third attempt if a red marble was obtained in the first attempt