தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்
ஆறாம் தவணைப் பரீட்சை - 2022

Conducted by Field Work Centre, Thondaimanaru.

6th Term Examination - 2022

FWC

Grade :- 13 (2022)	Combined Mathematics I- A	Time : 3 Hours 10 Minutes

Admission No						

Instructions

- This question paper consists of two parts; Part A (questions 1 - 10) and part B (questions 11 - 17).

Part - A

- Answer all questions. Answers should be written in the space provided on the questions paper. If additional space needed, you may use additional answer sheets.

Part - B

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- Only part B of the paper is allowed to be taken out of the examination hall.

Combined mathematicsI		
Part	**Question**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Combined Maths I	
Combined Maths II	
Total	
Final Marks	

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Part - A

1. Using the principle of mathematical mediation prove that
$1 + 2 + 3 + 4 + \cdots + 2n = n(2n + 1)$ for all $n \in \mathbb{Z}^+$

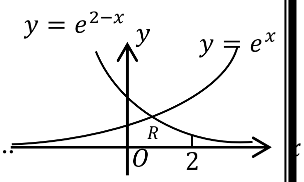
2. Sketch the graph of $y = x^2 - 4$, and $y = |x - 2|$ in the same diagram. Hence, find all the real values of x satisfying the inequality $2x^2 - 2 \geq |x - 1|$

3. Sketch, in an Argand diagram, the locus of the points that represent amplers number z satisfying $\left|z-1-i\sqrt{3}\right| = 1$. Hence, express the complex number z represented by the minimum of $Arg\, z$ at these loci in the form $r(\cos\theta + i\sin\theta)$ where $r > 0$, $0 < \theta < \frac{\pi}{2}$.

4. find the value of k, if the coefficient of term indepardment x in the binomial expansion is of $(1-kx^2)^2\left(1+\frac{1}{x}\right)^6$ is one.

5. Show that, $\lim_{x\to 0} \frac{1-\cos\frac{x}{2}}{\sqrt{x^3+x^2+4}-2} = \frac{1}{2}$

6. Show that the volume of the create solid of the region R rotate through 2π radian about X axis, enclosed by the curves $y = e^x$, $y = e^{2-x}$, $x = 0$, $y = 0$, and $x = 2$ is $\pi(e^2 - 1)$ cubic units.



7. Show that the point $P(a\cos^3\theta,\ a\sin^3\theta)$ on the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ where $0 < \theta < \frac{\pi}{2}$ and $a > 0$ and also show that $\frac{dy}{dx} = -\tan\theta$. Find the value of same parameter θ of the corresponding point P if the $-\frac{1}{\sqrt{3}}$ is the gradient at the tangent on the point P in the curve.

8. The equation of the sides PQ and PS are $3x + 4y - 1 = 0$ and $x - y + 3 = 0$ of the parallelogram $PQRS$ respectively. Find the equation of PR and SR, If $R = (2,1)$

9. Find the equation of the circle through the point $(0, 2)$ and touch the line $x + y = 0$ at origin.

10. Solve the equation $\sqrt{2}\,(\sin 2x - \sin x) + (2\cos x - 1) = 0$ for $0 < x < \frac{\pi}{2}$.

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6th Term Examination - 2022

Grade :- 13 (2022)	**Combined Mathematics – I - B**

Part - B

Answer only the five questions.

11. (a) let $f(x) = ax^2 + 2x + c,$ and $g(x) = bx^2 + x + c$ where given $a, b,$ and c are non zero real constant.

given $f(x),$ and $g(x)$ have a common root . α them show that $\alpha = \frac{1}{b-a}$.

find c in terms of a and b.

(i) If the discriminant Δ_1 of the equation $f(x) = 0$ then show that $\Delta_1 = \frac{4b^2}{(b-a)^2}$ Hence, show that the roots of $f(x) = 0$ are real and distinct.

(ii) If the discriminate Δ_2 of the equation $g(x) = 0$ the show that $\Delta_2 = \left(\frac{a-3b}{b-a}\right)^2$ Hence, If the roots of $g(x) = 0$ coincide them show that $a = 3b$.

(iii) Let β and γ are other roots of $f(x) = 0$ and $g(x) = 0$

show that $\beta = \frac{a-2b}{a(b-a)}$ and $\gamma = \frac{a-2b}{b(b-a)}$

(b) Let $h(x) = ax^3 + bx^2 + cx + 1,$ where a, b and c are real constant, If $x^2 - 4$ is a factor of $h(x)$ then show that $b = \frac{-1}{4}$ further given the remainder when $h(x)$ is divided by $x^2 - 1$ is $x + k$. Where k is a real constant. Find the values of a, b and k.

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12. (a) To set up a health committee in a school need to select 6 out of 2 male students, 2 female students, 2 male teachers, 2 female teachers, 1 male non – academic staff and 1 female non academic staff. Find the number of ways the team can choose in each of the following groups.

(i) If anyone can choose 6 people

(ii) If you want to select 3 men and 3 women.

(iii) If the all types of students, teacher and non – academic staff are to be included.

(iv) If you want to select three males and three females that can accommodate all types of students, teaches and non – academic staff

(b) Let$U_r = \frac{1}{(2r-1)(2r+1)} + 4r(r+1)$ and $V_r = \frac{1}{2(2r-1)} - \frac{4}{3}(r-1)r(r+1)$ for $r \in \mathbb{Z}^+$ show that . $V_r - V_{r+1} = U_r$ for $r \in \mathbb{Z}^+$ Hence, show that $\sum_{r=1}^{n} U_r = \frac{1}{2} - \frac{1}{2(2n+1)} + \frac{4}{3}n(n+1)(n+2)$ for $n \in \mathbb{Z}^+$ Is the infinite series $\sum_{r=1}^{\infty} U_r$ convergent? give the reason for your answer $W_r = \{r(r+2)\}^{(-1)^r}$ for $r \in \mathbb{Z}^+$ Deduce, find the sum of $\sum_{r=1}^{2n} w_r$.

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13. (a) Let $\boldsymbol{A} = \begin{pmatrix} a & 0 \\ 2 & -2 \\ b & 3 \end{pmatrix}, \boldsymbol{B} = \begin{pmatrix} 1 & 5 \\ 4 & -1 \\ 2 & 1 \end{pmatrix}, \boldsymbol{C} = \begin{pmatrix} 15 & 6 \\ c & 5 \end{pmatrix}$ are matrices such that $\boldsymbol{A^T B} = \boldsymbol{C}$ Where $a, b,$ and $c \in \mathbb{R}$. Show that $a = 1,\ b = 3$ and $c = 1$ write $\boldsymbol{C^{-1}}$ to the above values, Final the matrix $\boldsymbol{P}$ such that $\boldsymbol{C(P + 2I) = 3C + I}$ where I is the congruent matrix of order 2.

(b) Let $z,\ w \in \mathbb{C}$

Show that, $|z - 2i|^2 = |z|^2 - 4\,\mathrm{Im}(z) + 4$ and

$|1 + 2iz|^2 = 1 - 4\mathrm{Im}(z) + 4|z|^2$ deduce $\left|\frac{1+2iz}{z-2i}\right| = 1$ if and only $|z| = 1$ for $z \neq 2i$

Find the complex number z such that $\left|\frac{1+2iz}{z-2i}\right| = 1$ and $Arg(2iz) = \frac{\pi}{6}$.

(c) State the Demoivres theorem for index of positive integer. Express the complex number $\sqrt{6} + \sqrt{2}i$ in the form of $r(\cos\theta + i\sin\theta)$ where $r > 0$ and $0 < \theta < \frac{\pi}{2}$ Using the Demoivres theorem and show that $\left(\sqrt{6} + \sqrt{2}i\right)^6 = -512$

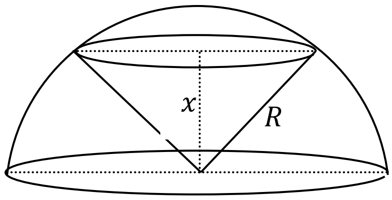
14. (a) Let $f(x) = \frac{x-2}{(x-1)^2}$ for $x \neq 1$

Show that derivative of $f(x)$ is $f'(x) = \frac{3-x}{(x-1)^3}$ for $x \neq 1$, hence. Final the interval of increasing and decreasing the function of $f(x)$ further find the coordinates, of turning points. Given the second derivative $f''(x) = \frac{2(x-4)}{(x-1)^4}$ of $f(x)$ for $x \neq 1$

Find the coordinates of point of inflection of the function $f(x)$

Stretch the rough diagram of $y = f(x)$ and indicate the asymptotes, turning points and inflection points.

(b) A right circular cone with an axis extending from a hemisphere solid to the centre of the sphere emerges as shown in the figure, the vertex of the cone corresponds to the centre of the sphere. If the height of the cone is x. show that the volume of the cone is given by $v = \frac{1}{3}\pi(R^2x - x^3)$. Where R is the radius of the sphere. Show that the volume of the cone cannot be more than $\frac{1}{3\sqrt{3}}$ times the volume of the sphere.



15. (a) The given the constants A and B are exists such that

$3x^2 + 7x \equiv A(x^2 + 4x + 5) + (x-1)(Bx + 5)$ and for all $x \in \mathbb{R}$. Final the value of A and B, hence write $\frac{3x^2+7x}{(x-1)(x^2+4x+5)}$ into partial fraction

Find the value of $\int \frac{3x^2+7x}{(x-1)(x^2+4x+5)} dx$.

(b) Using integration by parts, find $\int x^3 (\ln x)^2\, dx$

(c) Using the formula $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

Where a is constant given that $I = \int_0^{\frac{\pi}{2}} \cos^4 x \sin^2 x\, dx$ and

$J = \int_0^{\frac{\pi}{2}} \cos^2 x \sin^4 x\, dx$ Show that $I = J$

Further, show that $I = \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos^2 x \sin^2 x\, dx = \frac{\pi}{32}$.

16. The vertex of the rhombus $ABCD$ is given as $C \equiv (2, -3)$ and the diagonal AC is given by the line $l \equiv 2x + y - 1 = 0$ and AB is given by the line $l_1 \equiv 2x - y + 1 = 0$ diagonals AC and BD meet at E .

(i) Find the coordinates of E and A

(ii) Show that $x - 2y - 3 = 0$ as the equation of the straight line l_2 passing through the diagonal BD.

(iii) Show that any point between A and E on line l can be written in the form $(t,\ 1 - 2t)$ of the coordinates of a point P where $0 < t < 1$.

(iv) Show that $= \frac{5}{9}$, if the perpendicular distance from P to AB and BD are equal.

(v) Show that equation of the inner circle $S \equiv 9x^2 + 9y^2 - 10x + 2y - 6 = 0$ that touches the sides of the triangle ABD

(vi) Write the equation of the circle S' with diameter AB.

(vii) Are the circles S and S' intersecting at orthogonal.

17. (a) Prove that $\sin(x + y) + \sin(x - y) = 2\sin x \cos y$

using above prove,

show that $\frac{\sin x\,.\,\cos 3x}{\sin 3x\,.\,\cos x} = \frac{2\cos 2x - 1}{2\cos 2x + 1}$. By substituting the corresponding value to x.

prove that $\tan 15^0 = 2 - \sqrt{3}$.

further, it given $y = \frac{2\cos 2x - 1}{2\cos 2x + 1}$ show that $\cos 2x = \frac{y+1}{2(1-y)}$.

further show that $\frac{\sin x\,.\,\cos 3x}{\sin 3x\,.\,\cos x}$ does not exist between $\frac{1}{3}$ and 3 to all real value of x.

(b) State the sine rule in the usual notation to a triangle ABC

prove that $\frac{\sin A + \sin B}{\sin C} = \frac{a+b}{c}$ in the usual in natation of triangles ABC.

Further given $a + b = 2c,\ A - B = 90^0$, show that $\sin\frac{C}{2} = \frac{1}{2\sqrt{2}}$

(c) Solve $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}(2\sin^2 x)$.

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Conducted by Field Work Centre, Thondaimanaru.
6th Term Examination - 2022

Grade :- 13 (2022)	Combined Mathematics – II A	Time : Three hours Additional Reading Time – 10 minutes

Admissin No						

Instructions:

- **Part A** (Questions 1 - 10) and **Part B** (Questions 11 - 17).
- **Part A :**
  Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space needed.
- **Part B :**
  Answer five questions only.
- At the end of the time allotted, tie the answer scripts of the two pats together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- You are permitted to remove only Part B of the question paper from the Examination Hall.

For Examiners' Use only

(10) Combined Mathematics		
Part	**Question No.**	**Marks**
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
Total		

Combined Maths I	
Combined MathsII	
Total	
Final Marks	

Part - A

1. A particle of mass m is released from a height of h to fall on smooth horizontal plane. When it hit the ground and reversed, the loss of energy is $\frac{1}{4}mgh$, find the elastic eccentricity between the particle and the plane

2. A particle is projected with a velocity u from the top of a tower of height h , Show that the greatest range through the base of the tower is $\frac{u}{g}\sqrt{u^2 + 2gh}$.

3. A car of mass 100 kg moves on a horizontal path against a resistance force of $Kv \;\; N$. where $V \; ms^{-1}$ speed, K- constant. If the car moves with power of 40 kw at a speed of 20 ms^{-1}, Find the value of K. When the car moves upward on a path which makes 30° with horizontal of power 80 kw with speed 20 ms^{-1}, find the acceleration?

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4. A particle of mass m is suspended by an inextensible string of length a. When on equilibrium another particle of macs m collide with this particle horizontally with speed $2\sqrt{ag}$ and accombined. When the accombined particles comes to rest, find the angle formed by the string with downward vertical.

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5. Particles A and B of mass m, 2 m are joined with an inextensible string of length $2a$ placed on a smooth horizontal table with a gab a and gives a speed u to B in the direction perpendicular to AB, find the velocity of B just after the string tightens.

6. If the points O, A, B, C are such that $\overrightarrow{OA} = 5\underline{a}$, $\overrightarrow{OB} = 15\underline{b}$, $\overrightarrow{OC} = 24\underline{b} - 3\underline{a}$ then show that A, B, and C colinear points. Deduce the ratio of the point B devides AC.

7. A uniform rod AB of weight W and length 2a is hinged at A and can rotate about A. A string join at B is tied at C such that AC = AB and AC lies in vertical. By taking $A\hat{B}C = \theta$ find the tension on the string and the reaction at A by drawing the triangle of force.

8. A uniform rod is of length l and weight w. The rod is under limiting equilibrium such that one end rest on a rough ground and the other end rest on a peg which is at a height of 'a' from the ground. Show that the coefficient of friction between the ground and the rod is $a\sqrt{\frac{l^2-a^2}{l^2+a^2}}$.

9. In a road there are 20 houses in the left side and 10 houses in the right side of the road. In each sides half the houses has computers. When a house from the road is selected at random find the probability of that house lie in the right side of the road or has computer.

10. In a class the average marks obtained by girls for mathematics is 62 and the average marks obtained by boys for mathematics is 52. If the average marks obtained by whole students is 60, find the ratio between the boys and girls.

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6th Term Examination - 2022

FWC

தரம் :- 13 (2022) | **Combined Mathematics – II B**

11. (a) A particle P is projected vertically wpward from a point O, at a height 'h' above the ground level with velocity 4u. After $\frac{u}{g}$ time that P had thrown, the particte Q is dropped from O, in vertically downward under gravity. When P is at a height of $\frac{7u^2}{2g}$ above the point of projection after passing the greatest height, Q hit the ground. Draw the velocity time graph for the above two particles motion on the same diagram, upto both particles hitting the ground. From the velocity time graph.

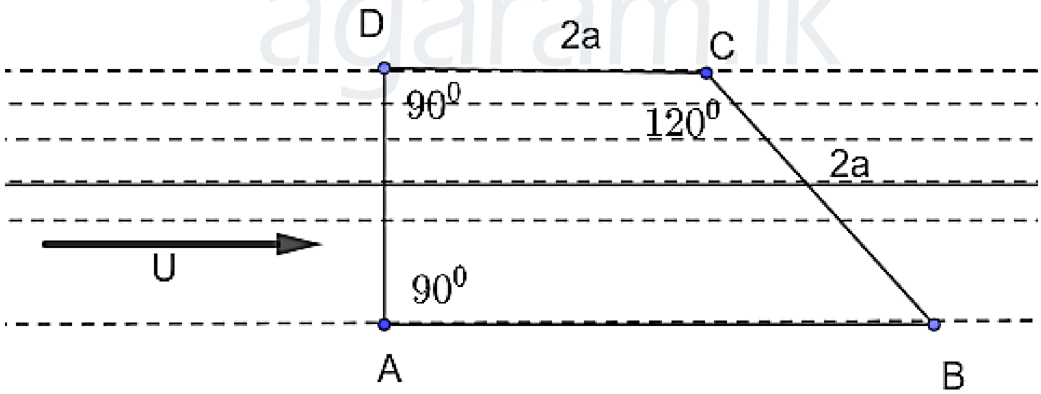
i) Find the seed of P, When Q is dropped to fall.

ii) When P reached the highest point. Find the speed of Q and the distance that Q moved.

iii) Find h in terms of u , and g.

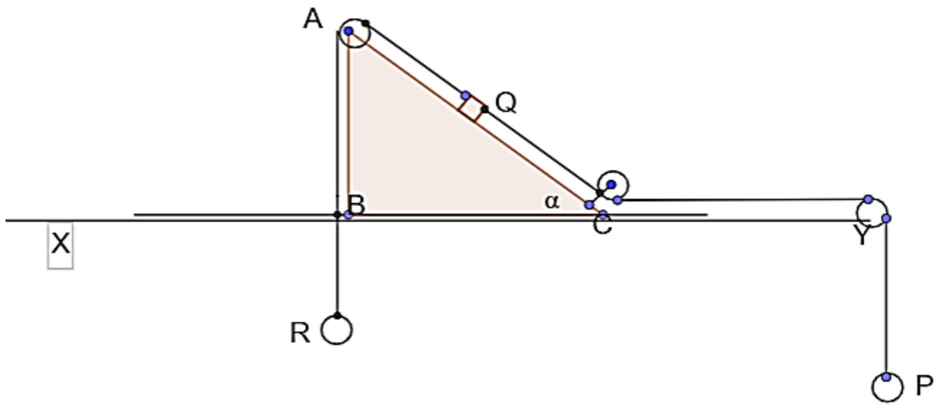
iv) What is the time taken to P to hit the ground?

(b)



In the diagram $ABCD$ shows four position on parallel banks of a river. Here AB is parallel to DC and A, B lie one side and D, C lie other side. $BC = CD = 2a$, $B\hat{C}D = 120^0$, $A\hat{B}C = 60^0$. River flows with a speed u in the direction parallel to $\overrightarrow{AB}$. Two men P and Q can swim with speeds $2U$ and $\sqrt{5}U$ respectively relative to the river. P swim from B to reach D. At the same time as P started, Q start to swim from $B$to A and then immediately returned to D from A. Using the principle of relative velocity by drawing velocity triangles of motions of P and Q separately, find the speeds of P and Q relative to earth and show that P reaches D at first.

12. (a)



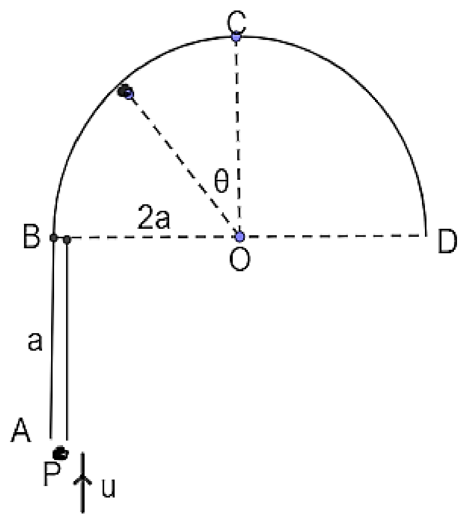
ΔABC is a triangle obtained by a verticarl cross section of the smooth wedge through its centre of gravity. Mass of the wedge is $4m$. $A\hat{B}C = 90^0$, $A\hat{C}B = \alpha$. The side BC of the wedge lies along a smooth straight gap on the table and B always lies in this gap. As shown in the diagram the particle P of mas $3m$ is joined to one end of a light inextensible string and passing over a light smooth pulley Y which is fixed at the edge of the table, then passing through a pully which is fixed at C on the wedge and the other end is joined to Q of mass 2m, which lies in the greatest slope of AC. At the same time one end of another inextensible string is joned to Q and passing over a pully which is fixed at the top A of the wedge and the other end carries a mass m at R through the gap. Initially the parts of the strings are hold tightly and released the system smoothly from rest. In the consequent motion, when all the particles are in motion.

i) Denote the forces act on the wedge and the particles

ii) Express to accelerations of the wedge and the particles clearly.

iii) Derive equations, sufficient for finding the tensions on the strings and accelerations of the wedge and the particles.

b) The diagram shows BCD is a vertical cross – section of a semi circular shaped spherical shell of centre O, radius $2a$ and fixed in vertical position. Here BD Horizontal and OC vertical. In B shown in the diagram a narrow smooth tube AB of length a is joined vertically. From A along the tube, a partied P of mass m is projected upward by speed u ($u > \sqrt{2ga}$). In the consequent motion, when the particle makes θ anticlockwise with OC.



i) Find the speed V of the particle.

ii) At this moment, find the reaction R on the particle by the spherical shell.

iii) If $u = 2\sqrt{2ag}$ then describe the motion of the particle.

13. One end of a light elastic string is attached to a point on the ceiling, and the other end carries freely a mass 2m, When in equilibrium the string extend two times its natural length, find the modulus of elasticity.

One end of a light elastic string of natural length 2a is attached to a point O on the ceiling and the other end carries a mass 2m. When in equilibrium it is at A of depth 6a below O. Using the above write the modulus of elasticity. When in equilibrium the velocity $3\sqrt{ag}$ is given to the particle in the direction of vertically downward. In the consequent motion when the particle is at a depth x below O $(x > 2a)$, Show that it satisfies the equation. $\ddot{x} = \frac{-g}{4a}(x - 6a)$.

If $X = x - 6a,$ show that the above equation satisfy the basic equation of simple harmonic motion, $\ddot{X} = \frac{-g}{4a}X$ and write its centre.

If the solution of this of this kinematic equation is of the form $\dot{X}^2 = \omega^2(b^2 - X^2)$ find $b,$ w and then write the amplitude of the simple harmonic motion. When the particle is at a depth 9a below O at B in moving downward, an impulse is given in the direction of vertically upward, the particle immediately moves upward with velocity $\frac{\sqrt{23ag}}{2}$

i) When the particle is at a depth $(y > 2a)$ below O show that it satisfies the equation $\dot{y}^2 = 2gy - \frac{g}{4a}(y - 2a)^2$ and express this in the form $\dot{Y}^2 = {\omega_1}^2(c^2 - Y^2)$ where $Y = y - 6a$ and ω_1, c are to be determined.

ii) Show that the motion of the particle is simple harmonic motion and write the centre and amplitude.

iii) Show that the least time taken to the particle from start moving to reach the ceiling is $2\sqrt{\frac{a}{g}}\left(\frac{11\pi}{12} - cos^{-1}\left(\frac{3}{4\sqrt{2}}\right) + 1\right).$

14. (a) Position vectors of A and B with respect to O are $\underline{a}$ and $\underline{b}$ respectively. Mid point of AB is $M.$ C is a point on OM such that $2.0C = CM.$ D is a point on OB such that $5\,OD = 3\,OB,$ DC Produced meet OA at $N.$

$NC = \mu CD \;\; ON = \lambda OA$

i) Find $\overrightarrow{OM}$, $\overrightarrow{OC}$, $\overrightarrow{OD}$, in terms of $\underline{a}$ and $\underline{b}$

ii) Find $\overrightarrow{ON}$, $\overrightarrow{NC}$ interms of $\lambda, \mu, \underline{a}$ *and* $\underline{b}$.

iii) Find λ and μ using vector addition.

iv) Find the ratio of N divides OA and the ratio of C divides ND.

(b) $ABCD$ is a rectangle with $AB = 8cm$ and $BC = 6cm$ $8, 4, 6, 5, 10, 5\ N$ are act along $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$, $\overrightarrow{AC}$, $\overrightarrow{BD}$ respectively.

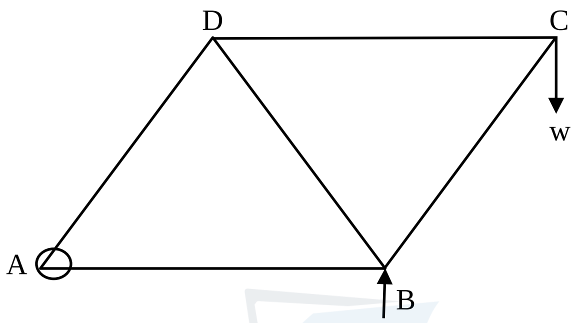
i) Find the magnitude and direction of the resultant.

ii) Calculate the point, that the line of action of the resultant intersect AB.

iii) Find the couple that should be added to pass the resultant through B.

15. (a) Two rods AB, and BC of equal length and equal weight are smoothly jointed at B. The end C is touching the rough horizontal plane and the end A is pivoted at a point above this plane. AB and BC makes α and β with the horizontal. Show that the coefficient of friction $\mu \geq \frac{I}{\tan\alpha + 3\tan\beta}$

(b)



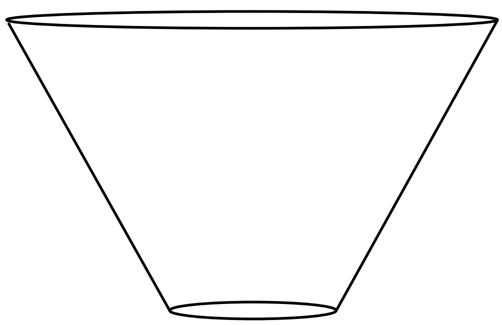
The frame work shown in the diagram is made by AB, BC, CD, DA and DB of light equal rods, hinged at A, supported at B and and a weight w is hung from C, AB is horizontal.

i) State the direction of force in the hinge A.

ii) using the Bow's notation draw a stress diagram and separate the stresses..

Find the reaction at the hinge A and the effect in the support B.

16. Using integration shorn that the centere of mass of a uniform hollow cone of height 'h' is lie along the axis of symmetry at a distance $\frac{h}{3}$ from the base.



From a unit area hollow right circular hollow cone of base radius $2a$ height $4a$ and mass per unit area ρ, a portion of right circular hollow cone of height a is removed, and the balance portion is shown in the diagram. Small circular base shown in the diagram is covered by a net shaped circular plate of mass $\frac{\sqrt{5}}{4}\pi a^2\rho$ and along the plane of the large circular brim end,. a rod (handle) of length $2a$ and mass which is two times the mass of the above net, is joined to it to make a tea strainer.

Find the centre of mass of the tea strainer from the centre of the open circular base, when the tea strainer is suspended from the end of the handle.

Find the angle the handle makes with vertical.

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17. (a) i) State Bay's theorem.

ii) Biscuits manufactured from a bakery, three persons A,B and C are packed in the percentage of 50%, 30% and 20% respectively. When packing these bisuits 3%, 2%, 1% respectively are damaged.

i) What is the probability of the packet of biscuit being damaged?

ii) If the packet of biscuits taken is damaged, find the probability of that the packet is packed by A.

(b) The marks obtained for mathematics for 100 students in a school is shown in the following table.

Marks	10–20	20–30	30–40	40–50	50–60	60–70	70–50
No of students	5	12	x	20	y	10	4

If the median marks is 44, find the values of x and y.

Calculate the mean and standard deviation of this distribution.