



**தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்  
நான்காம் தவணைப் பரீட்சை - 2022  
Conducted by Field Work Centre, Thondaimanaru.  
4<sup>th</sup> Term Examination - 2022**

Grade :- 13 (2022)

Combined Mathematics I- A

Time : 3 Hours 10 Minutes

Admission N<sup>o</sup>**Instructions**

- This question paper consists of two parts; Part A (questions 1 - 10) and part B (questions 11 - 17).

**Part - A**

- Answer all questions. Answers should be written in the space provided on the questions paper. If additional space needed, you may use additional answer sheets.

**Part - B**

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- Only part B of the paper is allowed to be taken out of the examination hall.

Combined mathematics I		
Part	Question	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
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	15	
	16	
	17	
	<b>Total</b>	

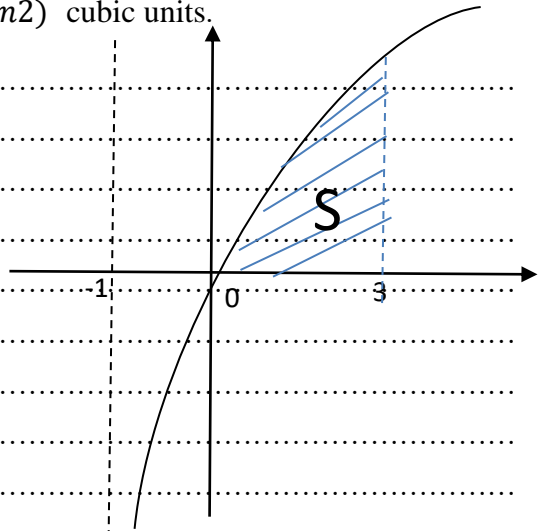
Combined Maths I	
Combined Maths II	
<b>Total</b>	
<b>Final Marks</b>	







7. Let  $s$  be the region endorsed by the curves  $y = \frac{x}{\sqrt{x+1}}$ ,  $y = 0$  and  $x = 3$ . Show that the area of  $s$  is  $\frac{8}{3}$  square units. Show that the volume of the region formed by the rotating the region  $s$  by  $2\pi$  radians about the  $x$  axis is given by  $\frac{\pi}{2} (3 - 4\ln 2)$  cubic units.



8. The straight line going through the fixed point  $(6, 3)$ , intersect the  $x$  - axis at A and Y axis at B, let R be the midpoint of AB, show that locus of R is  $3x + 6y - 2xy = 0$





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**4<sup>th</sup> Term Examination - 2022**

Grade :- 13 (2022)

Combined Mathematics I - B

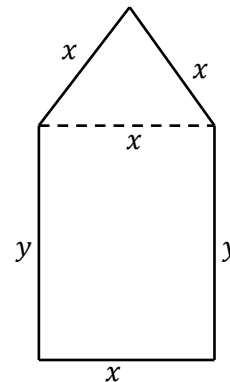
- 11) a) Let  $K > 2$ , show that the equation  $x^2 - 2kx + (k - 4)^2 = 0$  have distinct real roots. Let  $\alpha, \beta$  be the roots of the above equation. Write  $\alpha + \beta, \alpha\beta$  in terms of  $k$  and find the values of  $k$  such that both  $\alpha$  and  $\beta$  are positive. Now let  $2 < k < 4$ , show that the quadratic equation with the roots  $\sqrt{\alpha}$  and  $\sqrt{\beta}$  is  $x^2 - 2\sqrt{2}x + (4 - k) = 0$  and deduce the quadratic equation with the roots  $\frac{2}{\sqrt{\alpha}}$  and  $\frac{2}{\sqrt{\beta}}$ .
- b) If the equation  $ax^2 + bx + c = 0$  ( $a \neq 0$ ) and  $\acute{a}x^2 + \acute{b}x + \acute{c} = 0$  ( $\acute{a} \neq 0$ ) have the same roots show that  $\frac{a}{\acute{a}} = \frac{b}{\acute{b}} = \frac{c}{\acute{c}}$ . If the equation  $\alpha(x^2 - \alpha x + \beta) + \beta(x^2 - \beta x + \alpha) = 0$  and  $3x^2 - 10x + 8 = 0$  have the same roots. Show that the quadratic equation having the roots  $\alpha$  and  $\beta$  is  $x^2 - 6x + 8 = 0$ .
- 12) (a) Let  $f(x)$  be a polynomial of degree greater than one and  $a, b, c$  are distinct real constants. If the remainder when  $f(x)$  is divided by  $(x - a)(x - b)$  is  $px + q$ . Show that  $p = \frac{f(a) - f(b)}{a - b}$   $q = \frac{af(b) - bf(a)}{a - b}$ .
- If the coefficient of  $x$  in the remainder when  $f(x)$  is divided by  $(x - a)(x - b)$  and  $(x - a)(x - c)$  are equal then show that
- $$(b - c)f(a) + (c - a)f(b) + (a - b)f(c) = 0$$
- Let  $P(x)$  be,  $P(x) = 3x^4 + kx^3 + 2$ , If the coefficients of  $x$  in the remainders when  $p(x)$  is divided by  $x(x - 1)$  and  $x(x + 2)$  are equal. Show that  $k = 9$  also find the remainder and the quotient when  $P(x)$  is divided by  $x^2 + 1$ .
- (b) Let  $g(x) = 3x^2 - 6kx + 5k^2 - 2$ , where  $k$  is a real constant. Find the minimum value of  $g(x)$  in term of  $k$ . Hence,
- i) If the graph  $y = g(x)$  lies totally above the  $x$  axis. Find the values of  $k$ .
- ii) If the graph  $y = g(x)$  touches the  $x$  axis. Find the values of  $k$ .

13) (a) Let  $f(x) = \frac{(x-1)^3}{(x+1)^2}$  for  $x \neq -1$ .

Show that for  $x \neq -1$  derivative of  $f(x)$  is given by  $f'(x) = \frac{(x-1)^2(x+5)}{(x+1)^3}$ .

Hence find the range where  $f(x)$  is increasing and decreasing. Also find the co-ordinates of the turning point of  $f(x)$ . Find  $\lim_{x \rightarrow \pm\infty} f(x)$ , draw the rough sketch of the graph  $y = f(x)$  showing asymptotes, inflection point and y intercept. Using the graph find all real solution of  $x$  which satisfies  $f(x) < |f(x)|$ .

- (b) The window is in the shape of a triangle above a rectangle. The perimeter of the window is  $a$ . Show that for  $0 < x < \frac{a}{3}$ . The area  $A$  of window is given by  $A = \frac{1}{2} ax - \left(\frac{6-\sqrt{3}}{4}\right)x^2$ . Hence find the value of  $x$  for which the area  $A$  is maximum.



- 14) (a) Find the values of constants A, B and C. Such that

$$3x^4 + 2x^3 + 23x^2 + 7x + 40 \equiv Ax(x-1) + B(x-1)(x^2+4) + C(x^2+4)^2$$

Hence express  $\frac{3x^4+2x^3+23x^2+7x+40}{(x^2+4)^2(x-1)}$  as partial fraction and

Find  $\int \frac{3x^4+2x^3+23x^2+7x+40}{(x^2+4)^2(x-1)} dx$

- (b) Show that,

(i)  $\int_0^{\frac{\pi}{2}} \ln(\sin 2x) dx = \frac{1}{2} \int_0^{\pi} \ln(\sin x) dx$

(ii)  $\int_0^{\pi} \ln(\sin x) dx = 2 \int_0^{\frac{\pi}{2}} \ln(\sin x) dx$

Let  $I = \int_0^{\frac{\pi}{2}} \ln(\sin x) dx$ , for  $a$  is a constant using  $\int_0^a f(x) dx = \int_0^a f(a-x) dx$

Show that  $I = \int_0^{\frac{\pi}{2}} \ln(\cos x) dx$  also

Show that  $I = \frac{1}{2} \int_0^{\frac{\pi}{2}} \ln(\sin 2x) dx - \frac{\pi}{4} \ln 2$

Using the results (i) and (ii) above find the value of  $I$ .

Using integration by parts find the value of  $\int x \ln x dx$ .



- 15) (a) Show that the perpendicular distance from the line.  $ax + by + c = 0$  to the point  $(x_0, y_0)$  is given by  $\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$ .

In the rectangle ABCD,  $AB = 2AD$  and the line AB lies on the line  $x + 2y = 0$ . The diagonals of the rectangle ABCD intersect at the point  $E \equiv \left(\frac{5}{2}, \frac{5}{2}\right)$ . Show that the perpendicular distance from E to AB is  $\frac{3\sqrt{5}}{2}$ . Hence find the equation of other three sides of the rectangle.

- (b) The equation of diagonal AC of the rhombus ABCD is given by  $3x - y - 3 = 0$  and  $B \equiv (3, 1)$ . Also the equation of CD is given by  $x + ky - 4 = 0$ . Where k is a Constant. If  $D \equiv (\alpha, \beta)$ , Show that  $3\alpha - \beta + 2 = 0$ . Obtaining another relationship in terms of  $\alpha$  and  $\beta$ . Find the coordinates of point D. Hence show that  $k = 2$ , Find the equation of side AB.

- 16) (a) (i) Let the equation of two circles are

$$s \equiv x^2 + y^2 + 2gx + 2fy + c = 0$$

$$s' \equiv x^2 + y^2 + 2g'x + 2f'y + c' = 0 \text{ If the circles intersect orthogonally}$$

$$\text{show that, } 2\{gg' + ff'\} = c + c',$$

- (ii) Show that the equation of chord of contact to targets drawn from external points  $(x_0, y_0)$  is  $x_0x + y_0y + g(x_0 + x) + f(y_0 + y) + c = 0$  to the circle of  $s \equiv x^2 + y^2 + 2gx + 2fy + c = 0$  Show that the common equation of a circle  $s \equiv 0$  going through the point  $(1, 0)$  and having centre in the line  $x + y = 0$  is given by  $s \equiv x^2 + y^2 - 1 + \lambda(x - y - 1) = 0$  where  $\lambda$  is a parameter the circle  $s \equiv 0$  intersect the circle  $s_1 \equiv x^2 + y^2 + 2x - 2y - 11 = 0$  orthogonally. Show that  $s \equiv x^2 + y^2 - 4x + 4y + 3 = 0$ . Show that the chord of contact of the two targets drawn from the point  $P(0, 4)$  to the circle  $s \equiv 0$  is given by  $u \equiv 2x - 6y - 11 = 0$  Show that this equation of the circle going through the point  $(1, 1)$  and the point of intersection of  $s \equiv 0$  and  $U \equiv 0$  is given by  $s_2 \equiv 3(x^2 + y^2) - 10x + 6y - 2 = 0$  If the circumference of the circle  $s_2 \equiv 0$  is bisected by the circle  $s_3 \equiv x^2 + y^2 - 2x - 2y + c = 0$  then show that  $c = 2$

- 17) (a) Write  $\cos(A + B)$ ,  $\cos(A - B)$  in terms of  $\sin A$ ,  $\cos A$ ,  $\sin B$ , and  $\cos B$ . Hence show that  $\cos(A + B) \cos(A - B) = \cos^2 A - \sin^2 B$ .

From this deduce that,

(i)  $\cos 2A = \cos^2 A - \sin^2 A$

(ii)  $\cos^2 \frac{3\pi}{24} + \cos^2 \frac{5\pi}{24} + \cos^2 \frac{7\pi}{24} - 2 \sin^2 \frac{\pi}{24} - 2 \sin^2 \frac{3\pi}{24} = \frac{3 + \sqrt{3}}{2\sqrt{2}}$

- (b) State the sine rule for the triangle ABC in usual notation.

$$x \neq n\pi + \frac{\pi}{2} \text{ for } n \in \mathbb{Z}.$$

Show that  $\sin 2x = \frac{2 \tan x}{1 + \tan^2 x}$  and  $\cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$ .

In the usual notation in the triangle  $\sin 2B + \cos 2B = \frac{31}{25}$  and  $AB = 10\text{cm}$  are given, show that there are two such triangles and find  $\sin A$ , the length of BC and CA each of the triangles.

- (c) Solve the equation

$$\tan^{-1}(e^x) + \tan^{-1}(2e^x) = \frac{3\pi}{4}.$$



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**4<sup>th</sup> Term Examination - 2022**

Grade :- 13 (2022)

Combined Mathematics II - A

Three hours and ten minutes

Admission No

**Instructions**

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**Part - A**

- Answer all questions. Answers should be written in the space provided on the questions paper. If additional space needed, you may use additional answer sheets.

**Part - B**

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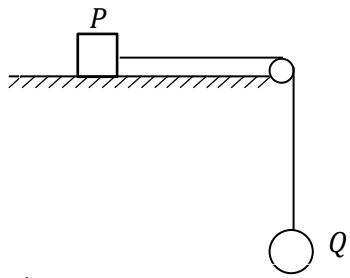
Combined mathematics I		
Part	Question	Marks
A	1	
	2	
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	9	
	10	
B	11	
	12	
	13	
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	<b>Total</b>	

Combined Maths I	
Combined Maths II	
<b>Total</b>	
<b>Final Marks</b>	





- 5)  $P, Q$  are two particles of masses  $12m, 6m$  respectively.  $P$  is on a rough horizontal table which is attached to one end of a light inelastic string which passes over a smooth pulley fixed on the edge of the table and carries  $Q$  while the string is taut, the system is released from rest if the velocities of the particles in the continuous motion at the time  $\sqrt{\frac{8a}{g}}$  seconds is  $\sqrt{2ag}$ . Find the coefficient of friction between  $P$  and the table and find the tension of the string.



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- 6) A particle  $P$  of mass  $m$  is attached to a light inelastic string of length  $5a$ .  $P$  is kept on a smooth horizontal table and the string is passed through a smooth hole on the table and carries a particle of mass  $m$ . Keeping the string taut the particle  $P$  moves with an angular velocity  $\omega$ . On the table while the particle  $Q$  moves with an angular velocity  $2\omega$ . Show that length of string below the table is  $2a$  and if  $\omega = \sqrt{\frac{g}{2a}}$ , Find the angle the string below the table makes with the vertical

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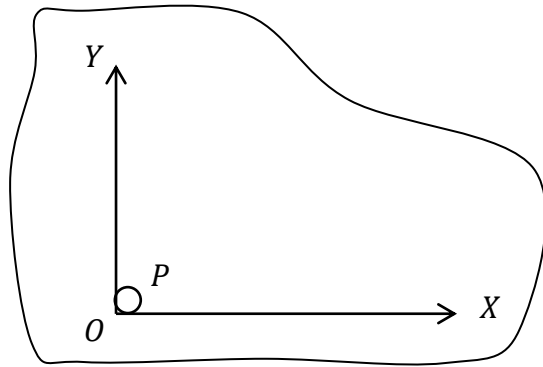
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7)



OX, OY are two axes perpendicular to each other a smooth plane,  $\underline{i}$  and  $\underline{j}$  are corresponding vectors along the X and Y axes respectively. When the particle of mass  $m$  is placed at O and given and forces of  $4\underline{i} - 5\underline{j}$  and  $x\underline{i} + y\underline{j}$  Newton, their resultant is found in the direction of  $\underline{i} - 2\underline{j}$ .

i) Find the angle between the resultant and  $\underline{j}$ .

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ii) Show that  $2x + y + 3 = 0$ .

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8) The position vectors of points A, B and C are  $\underline{a}$ ,  $\underline{b}$  and  $\underline{c}$  respectively.  $\alpha$ ,  $\beta$  and  $\gamma$  are non zero constants. If  $\alpha\underline{a} + \beta\underline{b} + \gamma\underline{c} = \underline{0}$  and  $\alpha + \beta + \gamma = 0$  then show that A, B and C are collinear.

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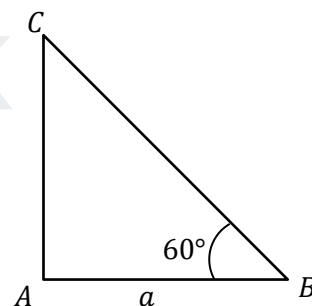
**தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்**  
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**Conducted by Field Work Centre, Thondaimanaru.**  
**4<sup>th</sup> Term Examination - 2022**

Grade :- 13 (2022)

Combined mathematics II - B

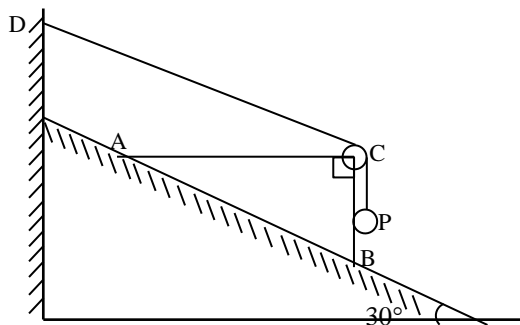
- 11) (a) A particle P is thrown vertically upwards with velocity  $V$  under gravity such that it reaches the maximum height  $\frac{25u^2}{2g}$  when the velocity of P is  $\frac{4V}{5}$  Upwards, another particle Q is thrown towards P with a velocity  $2u$  at a height  $h$  ( $h > \frac{9u^2}{2g}$ ) from the ground. If the velocity of Q when P, Q collide is  $5u$
- Show that  $V = 5u$
  - Draw the velocity time graph for P and Q in the same diagram from the time when P is thrown till the collision.
  - From the graph find the speed of P at the same time of collision (P and Q).
  - Find the height of the particle P from the land when the velocity of the particle P is  $\frac{4V}{5}$ .
  - Find time taken for Q to collide with P from the moment it was thrown.
  - Find the height of  $h$ .

- (b) A, B and C are three positions in a ground as shown in the figure P, Q and R are 3 racers in the positions A, B, C respectively. At the same time Q, R from the positions B, C respectively move in the direction AC with the uniform velocities. Mean while P with aim of catching up with Q first moves with the velocities  $2V$ . First he shakes hand with Q and immediately meet with the R moving with the same speed in a straight line.



- Using the principle of relative velocity draw the velocity triangle.
- Using the velocity triangle find the time taken for P to meet Q.
- Find the distance between P and R when P and Q meets.
- Find the time taken for P to meet R.
- Show that the time taken meet P and R is  $\frac{4a}{(\sqrt{13}-3)V}$

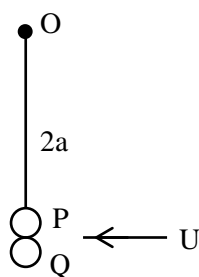
12)



The given figure shows vertical cross section through the centre of gravity of a smooth uniform wedge of mass  $4m$ . Where  $\hat{ACB} = 90^\circ$ . This wedge as shown in the figure is placed on an smooth inclined plan inclined to the horizontal such that the face  $BC$  is vertical and the face  $AB$  touches the inclined plane a particle of mass  $m$  is attached to a light inelastic string and the string passes through a smooth pulley in  $C$  and the other end is attached to  $D$  such that  $DC$  is parallel to  $AB$  as shown in the figure. The system is initially held such that the particle  $P$  is held close to  $B$  and face  $BC$  and is in the line of highest slope of the face and the system is released from the rest in the continuous motion, before  $P$  reaches  $C$ .

- Mark all the forces acting in the particle and wedge.
- Mark clearly the accelerations of the particle and the wedge.
- Gain enough equations to determine the accelerations of particle and the wedge, the tension in the string, the reaction given by the inclined plane to the wedge and the reaction between the wedge and the particle.
- If  $AC = 3a$ , the gain the equation to determine the time taken for  $P$  to reach  $C$ .

13)

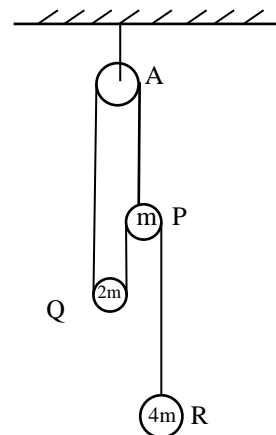


One end of a light inelastic string of length  $2a$  is attached to a point  $O$  which is at a height  $3a$  from ground and a particles of equal masses  $P, Q$  are joined to the other end and the particles are given the velocity  $u$  horizontally.

- In the continuous motion when the string makes an angle  $\theta$  with downward vertical. Find the velocity of particles  $P, Q$  and the tension in the string.
- If the particle  $P$  leaves smoothly from the particle  $Q$  when the string makes an angle  $\frac{\pi}{3}$  with the downward vertical then in the continuous motion. Find the velocity of  $P$  and the tension in the string when the particle  $P$  makes an angle  $\theta$  upward with the horizontal line passing through point  $O$ .

- (iii) Find the minimum velocity  $u_0$  of U for P to execute full circular motion.
- (iv) If the particles are initially given the velocity  $u_0$ , find the distance from the vertical line through O to the point where Q reaches the ground.

- 14) (a) As shown in the figure a light pulley A is fixed to a point in the ceiling and one end of a light inelastic string is attached to smooth uniform pulley P of mass  $m$ . The string passes above the pulley A and passes below a pulley Q of mass  $2m$  and again passes about the pulley P, and carries a particles R of mass  $4m$ . Initially the system is held such that the strings are taut and vertical and released from rest. In the continuous motion clearly mark the forces and acceleration of the particles, gain enough equations to determine the tension and accelerations of particles P, Q and R.

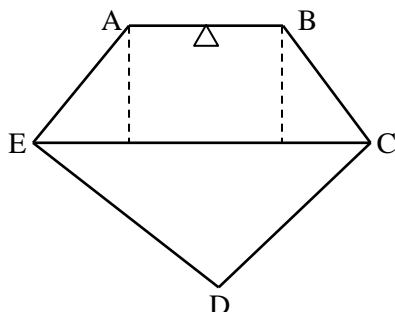


- (b) Two smooth spheres A and B of equal radii of masses  $km$  and  $m$  ( $2 < k < 3$ ) are projected towards each other with the velocity  $2u, 4u$  along the line joining the centres on a smooth floor. After the collision the speed of particle A is halved and the direction is reversed.
- (i) Find the velocity of B just after the collision in terms of  $k$ .
- (ii) If  $k$  is given as  $k = \frac{7}{3}$
- (a) Show that the velocity of B is inverted after the collision.
- (b) Find the coefficient of restitution and the impulse during the collision.

- 15) (a) The position vectors of A, B, C respectively to the point O are  $\underline{a}, 2\underline{b}$  and  $(\underline{a} + \underline{b})$  respectively the midpoint of OC is M. AM and OB meet at D the line drawn parallel to MA through O meet the extended CA at N. Let  $OD = \lambda OB, AN = \mu CA$  and  $AD = \gamma AM$ .
- (i) Find the vectors  $\overrightarrow{CA}$  and  $\overrightarrow{AM}$  in terms of  $\gamma, \lambda, \mu, \underline{a}, \underline{b}$ .
- (ii) Find the vector  $\overrightarrow{OD}, \overrightarrow{AN}$  and  $\overrightarrow{AD}$  in terms of  $\gamma, \lambda, \mu, \underline{a}, \underline{b}$
- (iii) Using appropriate vector additions find  $\lambda, \mu,$  and  $\gamma$ . Hence find the position vectors of D and N and find the ratio of the A divide the line CN.
- (b) In triangle ABC the midpoint of AC is D and  $AB = BD = DC = 2a$ . The forces of magnitude  $2P, \sqrt{3}P, P, 2P$  and  $P$  act along  $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{DB}, \overrightarrow{DC}$ , and  $\overrightarrow{DA}$  respectively.
- (i) Find the resultant in direction and magnitude of the system.
- (ii) Show that the line of action of resultant is perpendicular to BD.

- (iii) Find the point whose the line of action the resultant intersect AB.  
 (iv) Find the scense and magnitude of the couple that has to be added for the resultant to go through B.  
 (v) When the resultant in goes through B. if an additional force of  $6P$  is added through  $\overrightarrow{BD}$ . Find the direction of new resultant.

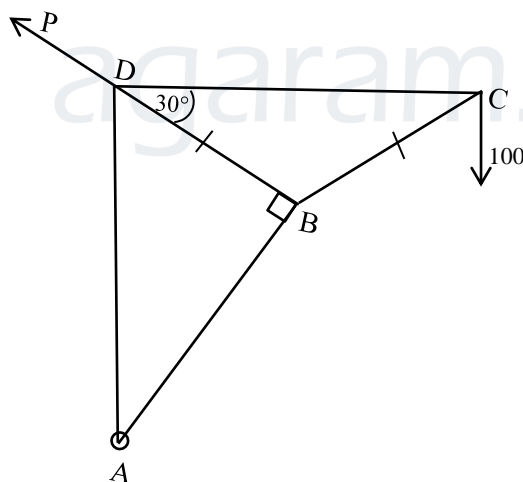
16) (a)



The uniform rods of weight  $W$  such that  $AB = AE = BC = 2a$ ,  $CD = DE = 4a$ . These are joined smoothly as shown in the figure and is kept in equilibrium by a wedge in the midpoint of AB and a light rod of length joined to the point C, E.

- (i) Find the reaction given by the wedge.  
 (ii) Find the stress in the light rod EC and the reaction in the joint D.  
 (iii) Find the horizontal and vertical components of the reaction in a point A in the rod AB.

(b)



The frame work made up of light rods is hinged at the joint A as shown in the figure. A force  $P$  act on D in the direction BD and a weight of  $100$  N is hanged at C So that DC is horizontal and AD is vertical.

- i) Find the direction of the reaction in the joint a using the bow notation draw the stress diagram and find the stress in each of the rods stating whether they are tension or thrust.  
 ii) Find the value of  $P$ .

17) (a) A smooth hemisphere container of radius  $\sqrt{3}a$  is fixed such that its curved surface touches the ground and its edge is horizontal inside the container rod AB of weight W and length  $6a$  is placed such that the end A is inside the hemisphere and the end B is on the outside. The point C on the rod touches the horizontal edge of the hemisphere. The rod is in equilibrium.

If the centre of gravity of the rod G is such that  $AG : GB = 1 : 2$

(i) Show that the inclination of the rod to the horizontal is  $\theta = \frac{\pi}{6}$ .

(ii) Find the reaction on A, C by the hemisphere.

(iii) If a particle of weight W is attached on the end B, show that the rod will never be in equilibrium.

(b) A rod of length  $2a$  and weight W is in limiting equilibrium in such a way that its one end is on a rough inclined plane and the other end is on a vertical smooth wall BD it makes an angle  $\theta$  with vertical. Let B is above A, If the coefficient of friction between the rod and the inclined plane is  $\lambda$ .

(i) Show that  $\tan \theta = 2 \tan(\lambda - \alpha)$

(ii) Find the reaction given by the wall to the rod.

(iii) Find the frictional force and reaction given by the inclined plane in the rod.

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